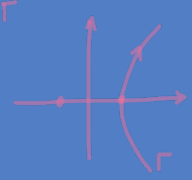


$$\begin{array}{ccc} \Psi & \xrightarrow{\text{oe}} & \tilde{\Psi} \in \mathbb{C}[[z^{-1}]] \\ \uparrow \mathcal{L}^0 & & \downarrow \mathcal{B} \\ \hat{\Psi} & \xleftarrow{\text{sum}} & \Psi \in \mathbb{C}\{t\} \end{array}$$

$$\left[\frac{\partial^2}{\partial y^2} - (m-1)y^{-1} \frac{\partial}{\partial y} - y^{n-2} \right] \psi = 0$$

$$Ai(y) = \int_{\Gamma} e^{\frac{t^3}{3} - yt} dt$$


$$1 + \frac{5}{6} z^{-1} + \frac{385}{216} z^{-2} + \frac{17017}{3888} z^{-3} + \dots$$

New perspectives on Borel summation

Two lectures by Aaron Fenyes (FMJH visitor) and Veronica Fantini (IHÉS)

18 and 20 July 2023

14:00–16:00

Amphithéâtre Léon Motchane
IHÉS, 35 Route de Chartres
91440 Bures-sur-Yvette



Overview

For certain problems, taking the Borel sum of an intuitively chosen divergent series solution will often produce a holomorphic solution. When does this happen, and what is special about the holomorphic solutions obtained in this way? We will present work in progress on these questions, focusing on two kinds of problems: linear ODEs and integrals over Lefschetz thimbles.

Day I

We will start the first day's lectures with a presentation of our questions and an overview of our expected results.

Next, we will review the Laplace and Borel transforms from a new perspective that highlights the geometric structure of the Borel plane. This geometric structure is relevant to both of the kinds of problems we study. For linear ODEs, the solutions that can be obtained by Borel summation are indexed by singular points on the Borel plane. Similarly, integrals over Lefschetz thimbles can be recast as Laplace integrals along rays departing from singular points.

Day II

The second day's lectures are dedicated to a family of examples: the *Airy-Lucas functions* introduced by Charbonnier et al.

(arXiv:2203.16523). These functions satisfy linear ODEs that generalize the Airy equation. They can also, like the Airy function, be expressed as thimble integrals. We will explain, from both perspectives, why these solutions can be obtained by Borel summation.

We will conclude by describing general classes of linear ODEs and 1d thimble integrals that can be analyzed in the same way.