

# **Workshop: "Topics on Stochastic Regularization"**

lundi 21 mars 2016 - mercredi 23 mars 2016

INSA Toulouse

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## Regularization by noise for the stochastic transport equation (Lisa Beck)

Auteur: Lisa Beck<sup>1</sup>

<sup>1</sup> *University of Augsburg*

We discuss several aspects of regularity and uniqueness for weak ( $L^\infty$ -) solutions to the (deterministic and stochastic) transport equation

$$du = b \cdot D u dt + \sigma Du \circ dW_t.$$

Here,  $b$  is a vector field (the drift),  $u$  is the unknown,  $\sigma$  is a real number,  $W_t$  is a Brownian motion, and the stochastic term is interpreted in the Stratonovich sense.

For the deterministic equation ( $\sigma = 0$ ) it is well-known that multiple solutions may exist and that solutions may blow up from smooth initial data in finite time if the drift is not regular enough. For the stochastic equation ( $\sigma \neq 0$ ) instead, it turns out that a suitable integrability condition (known from fluid dynamics

as the Ladyzhenskaya–Prodi–Serrin condition) on the drift is sufficient to prevent the formation of non-uniqueness and of singularities. After a short review of some techniques for the deterministic equation we explain how this regularization phenomenon, namely the conservation of Sobolev regularity of the initial data and the restoration of uniqueness, is obtained by means of PDE techniques (as opposed to stochastic characteristics). The results presented in this talk are part of a joint project with F. Flandoli, M. Gubinelli and M. Maurelli.

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## Regularization by stochastic drift (Pierre-Eric Chaudru de Raynal)

Auteur: Pierre-Eric Chaudru de Raynal<sup>1</sup>

<sup>1</sup> *Université de Savoie*

In this talk, we consider the well-posedness of stochastic differential equations of hypoelliptic type. It

is known from the pioneering work of Zvonkin that a stochastic microscopic perturbation regularizes

a differential system i.e., well-posedness holds for non-degenerate stochastic differential equations outside the classical Cauchy-Lipschitz framework.

Here, we consider a stochastic macroscopic perturbation of a differential system: the noise added in the system acts only by mean of the stochastic drift. Such a perturbation can be seen as a degenerate perturbation of hypoelliptic type. We show that the degenerate noise still regularizes the system (in the sense mentioned above) but, in comparison with the aforementioned work, a critical threshold appears for the regularity asking on the drift. This threshold can be seen as the price to pay to balance the degeneracy of the noise.

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## Restoration of uniqueness in mean-field games (François Delarue)

Auteur: François Delarue<sup>1</sup>

<sup>1</sup> *Université de Nice*

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## **Regularization by noise for transport and kinetic equations (Ennio Fedrizzi)**

**Auteur:** Ennio <sup>1</sup>

<sup>1</sup> *Ecole centrale de Marseille*

I will detail two cases where we have regularization for a PDE: the linear transport equation and a kinetic equation with force term. I will present some classical results for these two equations, related to well-posedness and regularity of solutions, that in the stochastic setting can be obtained under weaker assumptions on the regularity of the coefficients. These results are based on a careful analysis of the stochastic characteristics and the regularising properties of some associated parabolic/elliptic PDE.

These results are from joint works with Franco Flandoli, Enrico Priola and Julien Vovelle.  
This work is supported by LABEX MILYON / ANR-10-LABX-0070.

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## **Stochastic resonance (Samuel Herrmann)**

**Auteur:** Samuel Herrmann<sup>1</sup>

<sup>1</sup> *Université de Bourgogne*

The aim of the talk is to present a first step in studying stochastic resonance for huge systems of interacting particles. These systems can be compared to self-stabilizing diusions, i.e. processes attracted by their own law satisfying nonlinear stochastic differential equations (McKean-Vlasov). In this framework, several results are pointed out: the importance of the system inertia (exit problem), the large deviations behaviour of nonlinear diffusions in the small noise limit and the existence of several invariant measures (exactly three invariant measures in the selected model).

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## **Small noise limit of particle approximation for systems of conservation laws (Julien Reygner)**

**Auteur:** Julien Reygner<sup>1</sup>

<sup>1</sup> *Ecole des Ponts et Chaussées*

Parabolic systems of one-dimensional conservation laws (under diagonal form) can be approximated by systems of Brownian particles with a discontinuous drift coefficient. In order to approximate hyperbolic systems, it is of interest to determine the small noise limit of these particle systems. Under some uniform hyperbolicity assumption on the velocities of the particles, this task can be carried out and leads to a multitype version of the Sticky Particle Dynamics, which is a classical model

in astrophysics and in the study of gas dynamics. Without this uniform hyperbolicity assumption, some issues related with the persistence of randomness in the small noise limit arise.

Based on joint works with Benjamin Jourdain (<http://arxiv.org/abs/1307.0490>, <http://arxiv.org/abs/1501.01498>).