Structured Matrix Days

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The Parks-McClellan algorithm and Chebyshev-proxy rootfinding methods

Silviu Filip
École Normale Supérieure de Lyon
silviuioan.filip@ens-lyon.fr

During the 1970s, James McClellan and Thomas Parks developed an iterative routine for designing finite impulse response filters. Their work is based on the theory of minimax polynomial approximation, and today, it is one of the most well-known filter design methods in digital signal processing. In this talk, I will give insight into some recent work that I have been a part of, to improve the practical behavior of this algorithm [1]. My main focus will be on numerically stable root finding routines that rely on determining the eigenvalues of appropriate structured matrices [2]. They appear in the context of polynomial interpolation using the basis of Chebyshev polynomials. Extrema/root finding is usually the most computationally intensive part of the Parks-McClellan algorithm. Through interval subdivision techniques, we were able to construct a very robust implementation of this filter design routine, one that is more scalable than other existing implementations, like those found in Matlab and Scipy.

References

[1] Filip, S. *A robust and scalable implementation of the Parks-McClellan algorithm for designing FIR filters*, submitted for publication, preliminary version available at `https://hal.inria.fr/hal-01136005`

[2] Boyd, J. *Finding the Zeros of a Univariate Equation: Proxy Rootfinders, Chebyshev Interpolation, and the Companion Matrix*. SIAM Review 55, 2 (2013), 375396

Exploiting rank structures in the cyclic reduction

Dario A. Bini*, Stefano Massei†, Leonardo Robol†

*Dipartimento di Matematica, Università di Pisa, †Scuola Normale Superiore

bini@dm.unipi.it, stefano.massei@sns.it, leonardo.robol@sns.it

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Cyclic reduction is a useful tool that can be used for several purposes. It has been originally introduced by G.H. Golub and R.W. Hockney in the mid 1960s with the aim to solve certain linear systems, and has been generalized to solve nonlinear matrix equations defined by matrix power series. A survey of this algorithm and its applications is presented in [1].

An important application concerns the computation of the minimal nonnegative solution of the matrix equation $A + BX + CX^2 = 0$, encountered in Quasi Birth-Death (QBD) Markov chains, where A, B, and C are given $m \times m$ matrices and X is the matrix unknown. The cyclic reduction algorithm relies on the construction of the sequences

$$\begin{aligned} B_{i+1} &= B_i - A_i B_i^{-1} C_i - C_i B_i^{-1} A_i \\ A_{i+1} &= -A_i B_i^{-1} A_i, \qquad\qquad i = 1, 2, \ldots \\ C_{i+1} &= -C_i B_i^{-1} C_i \end{aligned} \tag{1}$$

where $A_1 = A$, $B_1 = B$, $C_1 = C$.

An important instance of this problem, which model double QBDs, is presented in [2] where the matrices A, B and C are tridiagonal and have a very large size. In some cases they are infinite. Under this assumption, the matrices generated by (1) lose their original structure and become dense matrices just after the first step. Apparently, the tridiagonal structure cannot be exploited and the cost of performing (1) becomes very large.

Fast Approximation of the Stability Radius and the H_∞ Norm for Large-Scale Linear Dynamical Systems

Michael L. Overton
Courant Institute, N.Y.U.
mo1@nyu.edu

Joint work with Nicola Guglielmi (L'Aquila),
Mert Gürbüzbalaban (M.I.T.) and Tim Mitchell (M.P.I.)

The stability radius and the H_∞ norm are well-known quantities in the robust analysis of linear dynamical systems with output feedback. These two quantities, which are reciprocals of each other in the simplest interesting case, respectively measure how much system uncertainty can be tolerated without losing stability, and how much an input disturbance may be magnified in the output. The standard method for computing them, the Boyd-Balakrishnan-Bruinsma-Steinbuch algorithm from 1990, is globally and quadratically convergent, but its cubic cost per iteration makes it inapplicable to large-scale dynamical systems. We present a new class of efficient methods for approximating the stability radius and the H_∞ norm, based on iterative methods to find rightmost points of spectral value sets, which are generalizations of pseudospectra for modeling the linear fractional matrix transformations that arise naturally in analyzing output feedback. We also discuss a method for approximating the real structured stability radius, which offers additional challenges. Finally, we describe our new public-domain MATLAB toolbox for low-order controller synthesis, HIFOOS (H-infinity fixed-order optimization — sparse). This offers a possible alternative to popular model order reduction techniques by applying fixed-order controller design directly to large-scale dynamical systems.

Quasiseparable Hessenberg triangular reduction for some diagonal plus low rank matrices

Leonardo Robol

Scuola Normale Superiore, Pisa

leonardo.robol@sns.it

We present a quasiseparable version of the classical Moler and Stewart's algorithm for the computation of the Hessenberg triangular form of a pencil $xA-B$. The classical algorithm computes two matrices H and T respectvely upper Hessenberg and upper triangular such that there exists two unitary matrices Q and Z for which $Q(xA-B)Z^* = xT - H$. This is usually the preliminary transformation carried out before applying the QZ iteration. We consider the particular case where $A = I + U_A V_A^*$ and $B = D_B + U_B V_B^*$ where D_B is real $n \times n$ diagonal matrix and U_A, U_B, V_A, V_B are rectangular $n \times k$ matrices with $k < n$. We provide a characterization of the quasiseparable structures of the partially reduced matrices obtained in the steps of a slight variant of the original algorithm and we propose an appropriate parametrization of these structures that leads to an asymptotic cost for the reduction of $O(n^2k)$ flops. We discuss the issues that arise in the implementation of a stable method for the Hessenberg triangular reduction and we present some numerical experiments. Some generalizations of the above setting are discussed, with examples of possible applications.

Computing the distance to the set of unstable quadratic matrix polynomials

Miloud Sadkane
Université de Brest
sadkane@univ-brest.fr

This talk considers the computation of the distance from a stable quadratic matrix polynomial to a nearest unstable one in the discrete sense. The distance problem is recast as a palindromic eigenvalue problem for which a structure preserving algorithm is developed.

Computation of Invariant Pairs and Matrix Solvents via Hankel Pencils

Esteban Segura Ugalde
XLIM-DMI, Université de Limoges
esteban.segura-ugalde@xlim.fr

We study the invariant pair problem for matrix polynomials. Invariant pairs extend the notion of eigenvalue-eigenvector pairs, providing a counterpart of invariant subspaces for the nonlinear case. They have applications in the numeric computation of several eigenvalues of a matrix polynomial; they also present an interest in the context of differential systems.

Here, we adapt the Sakurai-Sugiura moment method to the computation of invariant pairs, including some classes of problems that have multiple eigenvalues, and we analyze the behavior of the scalar and block versions of the method in presence of different multiplicity patterns. Results obtained via direct approaches may need to be refined numerically using an iterative method: we study and compare two variants of Newton's method applied to the invariant pair problem.

The matrix solvent problem is closely related to the invariant pair problem. Therefore, we specialize our results on invariant pairs to the case of matrix solvents, thus obtaining a moment-based computational approach.

Near optimal algorithms for structured matrices and for real and complex root refinement

Elias Tsigaridas
INRIA Paris-Rocquencourt
elias.tsigaridas@inria.fr

We combine some powerful techniques developed in the area of univariate root finding to devise new algorithms for refinement of isolated complex and real roots that have nearly optimal Boolean complexity. One of the main ingredients is multipoint evaluation, which in turn is closely related to fast computations with structured matrices. We present nearly optimal complexity bounds for almost all the basic operations with structured matrices.

Joint work with Victor Y. Pan.

Structured low-rank matrix completion with nuclear norm minimization: the case of Hankel and quasi-Hankel matrices

Konstantin Usevich
GIPSA-LAB, Université de Grenoble
konstantin.usevich@gipsa-lab.grenoble-inp.fr

Minimal rank completion of matrices with missing values is a difficult nonconvex optimization problem. A popular convex relaxation is based on minimization of the nuclear norm (sum of singular values) of the matrix. For this relaxation, an important question is when the two optimization problems lead to the same solution. In the literature, this question was addressed mostly in the case of random positions of missing elements and random known elements. Our focus lies on the completion of structured matrices with fixed pattern of missing values, in particular, on Hankel and quasi-Hankel matrix completion. The latter appears as a subproblem in the computation of symmetric tensor canonical polyadic decomposition. In the talk, we give an overview of these topics and report recent results on the performance of the nuclear norm minimization for completion of rank-r complex Hankel and quasi-Hankel matrices.