

BSDEs, Numerics and Finance

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Timetable

Monday

2:00	Arnaud Lionnet	<i>Time-discretization of monotone BSDEs with polynomial growth</i>
2:30	Géraldine Bouveret	<i>A Weak Discrete American-Type Stochastic Target Problem and its Application</i>
3:00	Kai Du	<i>On solvability conditions for backward stochastic Riccati equations</i>
3:30	Thomas Kruse	<i>BSDEs with singular terminal condition and applications to optimal trade execution</i>
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4:00	Coffee Break	
4:30	Ricardo Romo	<i>Indifference fee rate for variable annuities</i>
5:00	Ludovic Moreau	<i>Stability results for constrained FB-SDEs</i>
5:30	Olena Ragulina	<i>Optimal control by franchise and deductible amounts in the classical risk model</i>
6:00	Roxana Dumitrescu	<i>Double reflected BSDEs with jumps and generalized Dynkin games</i>

Tuesday

9:30	Samuel Cohen	<i>EBSDEs and spatially stable capacities for graphs</i>
10:00	Victor Fedyashov	<i>Ergodic BSDEs with jumps and time dependence</i>
10:30	Pierre-Yves Madec	<i>Ergodic BSDEs related to PDEs with Neumann boundary conditions</i>
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11:00	Coffee Break	
11:30	Paul-Eric Chaudru de Raynal	<i>A cubature based algorithm for forward and forward-backward stochastic differential equation of McKean-Vlasov type</i>
12:00	Camilo Andrés Garcia Trillos	<i>Numerical solution of multi-scale SDEs</i>
12:30	Ivo Mihaylov	<i>An explicit Euler scheme with strong rate of convergence for non-Lipschitz SDEs</i>
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13:00	Lunch	
3:00	Soumana Abdoulaye Hima	<i>BSDE driven by G-brownian motion</i>
3:30	Yiqing Lin	<i>Localization Methods for GSDEs</i>
4:00	Xiaolu Tan	<i>Martingale transport with full marginals constraint</i>
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4:30	Coffee Break	
5:00	Kihun Nam	<i>BSDEs, BSEs, and fixed points</i>
5:30	Asmerilda Hitaj	<i>Are the benchmarks equity strategies still valid for hedge fund portfolio allocation?</i>
6:00	Gechun Liang	<i>Optimal Switching at Poisson Random Intervention Times</i>

Wednesday

9:30	Elisa Mastrogiacomo	<i>Pareto optimal allocations and optimal risk sharing for quasiconvex risk measures</i>
10:00	Ludovic Tangpi	<i>Robust duality without reference measure</i>
10:30	Lukasz Szpruch	<i>Customized tamed numerical schemes for FBSDEs</i>
11:00	Coffee Break	
11:30	Christoph Mainberger	<i>Supersolutions of Convex BSDEs under Constraints: Minimality, Duality, Markov Property</i>
12:00	Rui Mu	<i>Bang-Bang Type Nash Equilibrium Point for Markovian Nonzero-sum Stochastic Differential Game</i>
12:30	Nguyen Tuyet Mai	<i>Greeks without Resimulation in Spatially Homogeneous Markov Chain Models of Portfolio Credit Risk</i>
13:00	Lunch	
3:00	Céline Labart	<i>Simulation of doubly reflected BSDEs with jumps and RCLL barriers</i>
3:30	Jean-Francois Chassagneux	<i>Numerical stability analysis of the Euler scheme for BSDEs</i>
4:00	Goncalo dos Reis	<i>Securitization and equilibrium pricing under relative performance concerns</i>
4:30	Coffee Break	
5:00	Thibaut Mastolia	<i>Density analysis of BSDEs</i>
5:30	Andrea Cosso	<i>Long time asymptotics for fully nonlinear Bellman equations: a Backward SDE approach</i>
6:00	Alexander Steinicke	<i>Malliavin differentiation of random functions with applications to Lévy driven BSDEs</i>
8:00	Conference Dinner	

Abstracts

Soumana Hima Abdoulaye (Université de Rennes 1)

BSDE driven by G-brownian motion

We provide existence and uniqueness results for the backward stochastic differential equations driven by a G-Brownian motion $(B_t)_{t \geq 0}$ in the form:

$$Y_t = \xi + \int_t^T g(s, Y_s, Z_s) d\langle B \rangle_s - \int_t^T Z_s dB_s - (K_T - K_t),$$

under quadratic growth of the generator g in Z and boundedness of the terminal condition ξ . We use the BMO property of the process Z to obtain some a priori estimates, which imply the uniqueness of the solution (Y, Z, K) in the case when it exists. The existence of the solution is obtained by a method similar to Galerkin approximation.

Géraldine Bouveret (Imperial College London)

A Weak Discrete American-Type Stochastic Target Problem and its Application

We study a stochastic target problem with a controlled probability of success on a set of deterministic dates. Proceeding as in [1] we can suitably increase the state space and the controls to reduce the problem to a more standard stochastic target one. More precisely we can reduce the problem to a problem of super-replication of a Bermudean option. However the increased controls are then valued in an unbounded set. Nevertheless we can deduce the related dynamic programming equation and study some important properties of the value function (convexity, monotonicity and continuity). We then apply our results to the so-called quantile hedging example of [2]. We can extend their result to the case where the constraint holds on a set of deterministic dates and find a pseudo-explicit solution under their framework of complete market. Some numerical results are also provided.

[1] Bouchard B., Elie R. and Touzi N., (2009). Stochastic target problems with controlled loss. *SIAM Journal on Control and Optimization*, 48 (5), pp. 3123-3150.

[2] Follmer H. and Leukert P., (1999). Quantile Hedging. *Finance and Stochastics*, 3, pp. 251-273.

Jean-François Chassagneux (Imperial College London)

Numerical stability analysis of the Euler scheme for BSDEs

In this talk, we study the qualitative behaviour of approximation schemes for Backward Stochastic Differential Equations (BSDEs) by introducing a new notion of numerical stability. For the Euler scheme, we provide sufficient conditions in the one dimensional and multidimensional case to guarantee the numerical stability. We then perform a classical VN stability analysis in the case of linear f and exhibits necessary condition to get stability in this case. Finally, we illustrate our results with numerical applications. This is a joint work with Adrien Richou (University of Bordeaux)

Paul-Éric Chaudru de Raynal (Université de Nice Sophia Antipolis)

A cubature based algorithm for forward and forward-backward stochastic differential equation of McKean-Vlasov type

In this talk, we will present an approximation of weakly coupled forward-backward McKean-Vlasov processes arising from control problem of a marked player in a mean field environment. The main characterization of this sort of process comes from the fact that the dynamic of the system depends on its own law. Our approach is based on cubature on Wiener space, in the terminology of Lyons and Victoir. We will show how such a strategy leads to the construction of two algorithms: a cubature tree, that approaches the law of the forward component, and, conditionally to this tree, an algorithm that approaches the values of the backward component. Such procedures are of order one (in term of the number of discretization steps) provided the coefficients of the system satisfy some smoothness assumptions. Moreover, when structural assumptions are made on the dependence of the coefficients with respect to the law, we show that the cubature tree can be parametrized in order to obtain an approximation of any order of the law of the forward component, and that the backward scheme can be modified in such a way that the convergence is of order two. This is a joint work with C.A. Garca Trillos (LJAD)

Samuel Cohen (Oxford University)

EBSDEs and spatially stable capacities for graphs

By considering the theory of Ergodic BDSEs, we can define nonlinear probabilities/capacities with useful spatial properties. Taking the simple case where space is defined to be a graph, we use EBSDEs based on Markov chains to construct these objects. In this talk we will discuss questions around existence, numerical calculation and some applications.

Andrea Cosso (Université Paris Diderot - Paris 7)

Long time asymptotics for fully nonlinear Bellman equations: a Backward SDE approach

We study the large time behavior of solutions to fully nonlinear parabolic equations of Hamilton-Jacobi-Bellman type arising typically in stochastic control theory with control both on drift and diffusion coefficients. We prove that, as time horizon goes to infinity, the long run average solution is characterized by a nonlinear ergodic equation. Our results hold under dissipativity conditions, and without any nondegeneracy assumption on the diffusion term. Our approach uses mainly probabilistic arguments relying on new backward SDE representation for nonlinear parabolic, elliptic and ergodic equations. This is a work in collaboration with M. Fuhrman and H. Pham.

Goncalo dos Reis (University of Edinburgh)

Securitization and equilibrium pricing under relative performance concerns

We investigate the effects of a finite set of agents interacting socially in an equilibrium pricing mechanism by mixing concepts of [Horst Pirvu DosReis 2010], [Espinosa Touzi 2013] and [Frei DosReis 2011]. A derivative written on non-tradable underlyings is introduced to the market and priced in a equilibrium framework by agents who assess risk using convex dynamic risk measures expressed by Backward Stochastic Differential Equations (BSDE). More than being exposed to financial and non-financial risk factors, the agents interact with their peers by way of a performance concern. The equilibrium analysis leads to systems of fully coupled multi-dimensional quadratic BSDEs. Within our proposed models we prove the existence and uniqueness of an equilibrium. We show that aggregation of risk measures is possible and that a (non-standard) representative agent exists. We analyze the impact of the problem's parameters in the pricing mechanism, in particular how the agent's concerns rates affects prices and risk perception.

Kai Du (ETH Zurich)

On solvability conditions for backward stochastic Riccati equations

This talk concerns a special class of matrix-valued quadratic BSDEs, called backward stochastic Riccati equations (SREs), arising in stochastic LQ optimal control problems. In a classical setting proposed by Bismut ('76), a complete solvability result was established by Tang ('03), depending on a definiteness assumption on certain coefficients. The general form, usually called indefinite SREs, releasing the definite condition but instead involving an algebraic constraint in addition to the backward equation, has not been solved completely, but only for several very special cases. In this talk, we give some novel sufficient conditions for the solvability of indefinite SREs, including a very practicable criterion, based on a new-defined notion named "subsolution". These results seemed to cover almost all existing ones on the solvability of indefinite SREs. Several examples will be presented to illustrate the results.

Roxana Dumitrescu (Université Paris-Dauphine)

Double reflected BSDEs with jumps and generalized Dynkin games

We study double barrier reected BSDEs (DBBSDEs) with jumps and RCLL barriers, and their links with generalized Dynkin games. We provide existence and uniqueness results and prove that for any Lipschitz driver, the solution of the DBBSDE coincides with the value function of a game problem, which can be seen as a generalization of the classical Dynkin problem to the case of g -conditional expectations. Using this characterization, we prove some new results on DBBSDEs with jumps, such as comparison theorems and a priori estimates. We then study DBBSDEs with jumps and RCLL obstacles in the Markovian case and their links with parabolic partial integro-differential variational inequalities (PIDVI) with two obstacles. (joint work with M.C.Quenez and A. Sulem)

Victor Fedyashov (University of Oxford)

Ergodic BSDEs with jumps and time dependence

We look at ergodic BSDEs in the case where the forward dynamics are given by a solution to a non-autonomous (time-periodic coefficients) Ornstein-Uhlenbeck SDE with Lévy noise, taking values in a separable Hilbert space. We establish the existence of a unique bounded solution to an infinite horizon discounted BSDE. We then use the vanishing discount approach together with coupling techniques to obtain a Markovian solution to the EB-SDE. We also prove uniqueness under certain growth conditions. Applications are then given, in particular to risk-averse ergodic optimal control and power plant evaluation under uncertainty.

Camilo Andrés García Trillos (Université Nice Sophia Antipolis)

Numerical solution of multi-scale SDEs

In this talk, we present a numerical algorithm for approximating the solution of a strongly oscillating SDE, i.e. a system in which some ergodic state variables evolve quickly with respect to the other variables. The algorithm profits from homogenization results and consists of an Euler scheme for the slow scale variables coupled with a decreasing step estimator for the ergodic averages of the fast variables. We will also briefly discuss some techniques to improve the convergence rate of the basic algorithm.

Asmerilda Hitaj (University of Milano Bicocca)

Are the benchmarks equity strategies still valid for hedge fund portfolio allocation?

In the equity world different benchmarks (such as equally weighted (EW), global minimum variance (GMV), equally risk contribution (ERC) and maximum diversified ratio (MDR)) have been proposed in alternative to the cap-weighted index. These new approaches have attracted the attention of equity managers, as different empirical analysis demonstrate the superiority of these strategies with respect to cap-weighted and to strategies that consider both mean and variance. In this paper we focus our attention on hedge fund portfolios and analyze if the results obtained in the equity world are still valid. We consider 4 different hedge fund portfolios and the approaches used for portfolio selection are EW, GMV, ERC, MDR, mean-variance (MV), mean-variance-skewness (MVS: which is known to be a good model in case of hedge funds). For the MV and MVS approach we have used the multi objective approach (MOA) considering different weights for moments and co-moments. Moreover, we analyze the impact of weights constraints and cardinality constraints in the out-of-sample period for all the strategies under consideration.

Thomas Kruse (Université d'Évry)

BSDEs with singular terminal condition and applications to optimal trade execution

We provide a probabilistic solution of a not necessarily Markovian control problem with a state constraint by means of a Backward Stochastic Differential Equation (BSDE). Such a control problem arises for example when economic agents have to close a position in a market with a stochastic price impact. The novelty of our solution approach is that the BSDE possesses a singular terminal condition. We prove that a solution of the BSDE exists, thus partly generalizing existence results obtained by Popier 2006, 2007. We perform a verification and discuss special cases for which the control problem has explicit solutions.

Céline Labart (Université de Savoie)

Simulation of doubly reflected BSDEs with jumps and RCLL barriers

We study a discrete time approximation scheme for the solution of a doubly reflected Backward Stochastic Differential Equation (DBBSDE) with jumps, driven by a Brownian motion and an independent compensated Poisson process. Moreover, we suppose that the obstacles are right continuous and left limited (RCLL) processes with predictable and totally inaccessible jumps and satisfy Mokobodski's condition. Our main contribution consists in the construction of an implementable numerical scheme, based on two random binomial trees and the penalization method, which is shown to converge to the solution of the DBBSDE. Finally, we illustrate the theoretical results with some numerical examples in the case of general jumps.

Gechun Liang (King's College London)

Optimal Switching at Poisson Random Intervention Times

Optimal switching is the problem of determining an optimal sequence of stopping times for a switching system, which is often modeled by a stochastic process with several regimes. In this talk, we introduce and solve a new class of optimal switching problems, where the player is allowed to switch at a sequence of random times generated by an exogenous Poisson process, and the underlying switching system is governed by multidimensional penalized backward stochastic differential equation (BSDE). We show that the value of this optimal switching problem is nothing but the solution of the multidimensional penalized BSDE. The basic observation comes from the optimal stopping time representation for the one dimensional penalized BSDE. Then in a Markovian setting with one dimensional geometric Brownian motion and two regimes, we give a complete description of the structure of switching regions. The basic observation is that we can consider the difference of the value functions for the two switching regimes, and then employ the comparison principle for the one dimensional equation.

Yiquing Lin (Universität Wien)

Localization Methods for GSDEs

In this paper, stochastic differential equations driven by G-Brownian motion (GSDEs) with local Lipschitz conditions are discussed on the space $M_p^w([0, T]; \mathbb{R}^n)$ via a localization method. The existence and uniqueness result has been obtained and the stability of solutions has been considered.

Arnaud Lionnet (Oxford Man Institute)

Time-discretization of monotone BSDEs with polynomial growth

We consider the time-discretization schemes for BSDEs when the driver has polynomial growth in the primary variable Y and is monotone (decreasing). These drivers arise typically when considering reaction-diffusions PDEs, where the nonlinearity is often a polynomial in Y . The main finding is that, unlike for Lipschitz drivers, the explicit scheme can explode while the implicit scheme converges. By looking at the theta-schemes (which are mixed implicit and explicit schemes), we obtain through a Fundamental Lemma a global error estimate that reveals the more detailed picture. All schemes but the purely implicit one lack strict stability. However, schemes that are predominantly implicit do converge. As a by-product of our study, we can prove a higher rate of convergence for the trapezoidal scheme (equally implicit and explicit) under typical conditions. This is based on joint work with Gonalo dos Reis and Lukasz Szpruch.

Pierre-Yves Madec (Université de Rennes 1)

EBSDEs related to PDEs with Neumann boundary conditions

We study a class of ergodic BSDEs related to PDEs with Neumann boundary conditions. The randomness of the driver is given by a forward process under weakly dissipative assumptions with an invertible and bounded diffusion matrix. Furthermore, this forward process is reflected in a convex subset of \mathbb{R}^d not necessarily bounded. We show the link of such EBSDEs with PDEs.

Christoph Mainberger (TU Berlin)

Supersolutions of Convex BSDEs under Constraints: Minimality, Duality, Markov Property

We study supersolutions of backward stochastic differential equations under constraints on the class of admissible controls, a setting that corresponds to non-standard superhedging problems in incomplete financial markets. This framework in particular comprises classical constraints such as short-selling- or γ -constraints. More precisely, controls are restricted to continuous semimartingales of the form $dZ = \Delta dt + \Gamma dW$, while the generator in addition depends on the decomposition (Δ, Γ) , thereby incorporating penalizations of rapid changes of control values observed for instance in high-frequency trading. Assuming the generator to be positive, convex and lower semicontinuous, and to satisfy a specific growth condition in Δ and Γ , we prove the existence of a supersolution that is minimal at finitely many times and derive stability properties of the non-linear operator that maps terminal conditions to the time zero value of this minimal supersolution such as monotone convergence, the Fatou property and L^1 -lower semicontinuity. By means of the stability, we provide duality results within the present framework. We characterize the conjugate in terms of the decomposition parts of the controls and show that it is always attained. Using the dual problem we establish conditions for the existence of solutions of the associated BSDE under constraints. For the particular case of a quadratic generator we explicitly compute the conjugate by means of classical calculus of variations methods. Finally, we provide results on Markovian supersolutions within this framework. (Based on joint works with Samuel Drapeau, Gregor Heyne, Michael Kupper, and Ludovic Tangpi)

Elisa Mastrogiacomo (University of Milano Bicocca)

Pareto optimal allocations and optimal risk sharing for quasiconvex risk measures

Pareto optimal allocations and optimal risk sharing for coherent or convex risk measures as well as for insurance prices have been studied widely in the literature. In particular, Pareto optimal allocations have been characterized by applying inf-convolution of risk measures and convex analysis. In the recent literature, an increasing interest has been devoted to quasiconvex risk measures, that is risk measures where convexity is replaced by quasiconvexity and cash-additivity is dropped. The main goal of this paper is then to generalize the characterization of Pareto optimal allocations known for convex risk measures (see, among others, Jouini et al. (2008)) to the quasiconvex case. Following the approach of Jouini et al. (2008) for convex risk measures, in the quasiconvex case we provide sufficient conditions for allocations to be (weakly) Pareto optimal in terms of exactness of the so-called quasiconvex inf-convolution. Moreover, we give a necessary condition for weakly optimal risk sharing that is also sufficient under cash-additivity of at least one between the risk measures.

Thibaut Mastrolia (Université Paris Dauphine)

Density analysis of BSDEs

In this talk, we study the existence of densities (with respect to the Lebesgue measure) for marginal laws of the solution (Y, Z) to a quadratic growth BSDE appearing in many financial problems. Our approach relies on the use of Malliavin calculus. Besides, using the (by now) well-established connection between these equations and their associated semi-linear PDEs, together with the Nourdin-Viens formula, we provide estimates on these densities. This talk is based on a joint work with Dylan Possamaï (CEREMADE) and Anthony Réveillac (CEREMADE).

Ivo Mihaylov (Imperial College London)

An explicit Euler scheme with strong rate of convergence for non-Lipschitz SDEs

We consider the approximation of stochastic differential equations (SDEs) with non-Lipschitz drift or diffusion coefficients. We present a modified explicit Euler-Maruyama discretisation scheme that allows us to prove strong convergence, with a rate. Under some regularity conditions, we obtain the *optimal* strong error rate of $1/2$. We consider SDEs popular in the mathematical finance literature, including the Cox-Ingersoll-Ross, the $3/2$ and the Ait-Sahalia models, as well as a family of mean-reverting processes with locally smooth coefficients.

Ludovic Moreau (ETH Zürich)

Stability results for constrained FBSDEs

We are interested in this paper in the super-replication of a contingent claim under convex delta constraints. More specifically, we study the continuity of this price with respect to both time and the initial position of the underlying asset. We prove under Lipschitz-type assumptions local $1/2$ -Hölder continuity with respect to time and local Lipschitz continuity with respect to space. Additionally, we prove that the super-replication price under constraints coincide with the superreplication price of a suitably defined facelifted payoff. This is a joint work with Bruno Bouchard and Romuald Elie.

Rui Mu (Université du Maine)

Bang-Bang Type Nash Equilibrium Point for Markovian Nonzero-sum Stochastic Differential Game

In this work, we solve a Nonzero-sum stochastic differential game (NZSDG) with bang-bang type equilibrium controls by using backward stochastic differential equations (BSDEs). The generator is multi-dimensional and discontinuous with respect to z .

Kihun Nam (Princeton University)

BSDEs, BSEs, and fixed points

In this presentation, we will generalize BSDEs into Backward Stochastic Equations (BSEs): $Y_t + F_t(Y, M) + M_t = \zeta + F_T(Y, M) + M_T$. Then, we will show that there is a one-to-one correspondence between the solutions of the above BSE and the fixed points of the mappings determined by ζ and F . Using Banach fixed point theorem and Krasnoselskii fixed point theorem, we will show the existence and the uniqueness of solution for BSEs and BSDEs. In particular, novel existence results will be provided for (solution) path-dependent BSDEs and multidimensional quadratic mean-field BSDEs. This is a joint work with Patrick Cheridito.

Olena Ragulina (University of Kyiv)

Optimal control by franchise and deductible amounts in the classical risk model

The talk deals with the problem of survival probability maximization in the classical risk model when an insurance company has the opportunity to choose franchise and deductible amounts continuously. A franchise is a provision in the insurance policy whereby the insurer does not pay unless damage exceeds the franchise amount, whereas, deductible is a provision whereby the insurer pays any amounts of damage that exceed the deductible amount. As a rule, these provisions are applied when the insured's losses are relatively small to deter the large number of trivial claims. Moreover, a deductible stimulates the insured to take more care of the insured property. Normally, a franchise and a deductible imply also reduction of insurance premiums. Thus, changes in claim and premium sizes have an influence on the survival probability of an insurance company. Our problem is to maximize the survival probability adjusting franchise or deductible amounts. We apply stochastic control theory to solve this problem. We derive the Hamilton-Jacobi-Bellman equation for the optimal survival probability and prove the existence of a solution of this equation with certain properties. The verification theorem gives the connection between this solution and the optimal survival probability. Then we concentrate on the case of exponentially distributed claim sizes. We prove that we can always increase the survival probability adjusting the franchise amount in this case. We also show that the survival probability cannot be increased adjusting the deductible amount for exponentially distributed claim sizes.

Ricardo Romo Romero (Université d'Évry)

Indifference fee rate for variable annuities

In this paper, we work on indifference valuation of variable annuities and give a computation method for indifference fees. We focus on the guaranteed minimum death benefits and the guaranteed minimum living benefits and allow the policyholder to make withdrawals. We assume that the fees are continuously payed and that the fee rate is fixed at the beginning of the contract. Following indifference pricing theory, we define indifference fee rate for the insurer as a solution of an equation involving two stochastic control problems. Relating these problems to backward stochastic differential equations with jumps, we provide a verification theorem and give the optimal strategies associated to our control problems. From these, we derive a computation method to get indifference fee rates. We conclude our work with numerical illustrations of indifference fees sensibilities with respect to parameters.

Alexander Steinicke (University of Innsbruck)

Malliavin differentiation of random functions with applications to Lévy driven BSDEs

Let $X = (X_t)_{t \in [0, T]}$ be a Lévy process with Lévy measure ν and Lévy-Itô decomposition

$$X_t = \gamma t + \sigma W_t + \int_{(0, t] \times \{1 < |x|\}} x N(ds, dx) + \int_{(0, t] \times \{0 < |x| \leq 1\}} x \tilde{N}(ds, dx),$$

where $\gamma \in \mathbb{R}, \sigma \geq 0$ are constants, (W_t) denotes the Brownian part, N and \tilde{N} are the Poisson random measure and the compensated Poisson random measure, respectively. We consider the BSDE

$$Y_t = \xi + \int_t^T f(X^s, s, Y_s, Z_s, \bar{U}_s) ds - \sigma \int_t^T Z_s dW_s + \int_{(t, T] \times \mathbb{R}} U_{s,x} \tilde{N}(ds, dx),$$

where $X^s := (X_u)_{u \in [0, s]}$ and $\bar{U}_s = \int_{\mathbb{R}} g(U_{s,x}) g_1(x) \nu(dx)$ with g Lipschitz and differentiable and $g_1 \in L_2(\mathbb{R}, \mathcal{B}(\mathbb{R}), \nu)$. By a Malliavin differentiation formula for $f(X^s, s, Y_s, Z_s, \bar{U}_s)$, we describe sufficient conditions on the function f , such that if $\xi \in \mathbb{D}_{1,2}$ one may differentiate the BSDE in the sense of Malliavin calculus. Malliavin differentiation of this BSDE is important to access the Z process explicitly from the Y process. Moreover it is an essential tool for investigating regularity properties of BSDEs.

Lukasz Szpruch (University of Edinburgh)

Customized tamed numerical schemes for FBSDEs

In this talk we introduce a family of explicit numerical approximations for the forward backward stochastic differential equations (FBSDEs) with, possibly, no-globally Lipschitz coefficients. We will first focus on the forward process. We show that for a given Lyapunov function $V : \mathbb{R}^d \rightarrow [1, \infty)$ we can construct a suitably tamed Euler scheme that preserves so called V-stability property of the original SDEs. V-stability condition plays a crucial role in numerous stability and integrability results for SDEs developed by Khasminski. We will further show that developed methodology naturally extends to the time-discretizations of BSDEs. We show that newly developed methodology allows to analyse BSDEs with drivers having polynomial growth and that are also monotone in the state variable. Proposed schemes preserve qualitative properties of the solutions to the FBSDEs for all ranges of time-steps.

Ludovic Tangpi (University of Konstanz)

Robust duality without reference measure

The problem of dual representation of convex risk measures on a probability space is well understood. In this talk, we present a dual representation result for risk measures without assuming a reference probabilistic model. Our representation result is based on the topological structure of the sample space and the Riesz representation theory. As an application, we discuss the fundamental theorem of asset pricing in a robust non-dominated setting. (Based on Joint works with Patrick Cheridito and Michael Kupper)

Xiaolu Tan (University of Paris Dauphine)

Martingale transport with full marginals constraint

We extend the martingale version of the one-dimensional Bernier's theorem (Frechet-Hoeffding coupling) established in Beiglbock and Juillet (2012), Henry-Labordere and Touzi (2013) to the infinitely-many marginals case. By approximation technique, we show that for a class of cost/reward function, the solution of the martingale transport problem given infinitely-many marginals is provided by a pure downward jump local Levy model. In particular, it provides a new construction of the martingale peacock process, and a new remarkable example of discontinuous fake Brownian motion. Moreover, we also provide a duality result together with a dual optimizer in explicit form. This is a joint work with P. Henry-Labordere and N. Touzi.

Nguyen Tuyet Mai (University of Évry)

Greeks without Resimulation in Spatially Homogeneous Markov Chain Models of Portfolio Credit Risk

In this paper, we model credit portfolios by continuous-time Markov chains with some form of spatial homogeneity, so that direct Monte Carlo Greeks estimates, without resimulation, can be derived. We implement our results in two specific credit models: a common-shock model where the spatial homogeneity is straightforward, and a homogeneous group model where spatial homogeneity can be recovered by a change of measure and tools of Malliavin calculus. Our estimates are then extended to Greeking nonlinear prices given as solutions to Markov chain related BSDEs. The direct Monte Carlo Greek estimates are competitive with previously developed simulation/regression estimates, but they are also unbiased, and there is some evidence that they would be less impacted by the curse of dimensionality.